Derivation of K_d from a

In what follows we have left off the dependence on wavelength. In addition we have left off effects due to inelastic scattering.

Modeling $K_d(z)$ requires the inclusion of the depth dependence of the shape of the radiance distribution. This can be accomplished by using Gershun's equation:

$K(z) = a(z) / \bar{\mu}(z)$

where a(z) is the absorption coefficient and $\bar{\mu}(z)$ is the average cosine of the light field. $K_d(z)$ differs from K(z) by only a few percent, so that we may set:

$K_d(z) \approx a(z) / \bar{\mu}(z)$.

Berwald et al. (1995) have derived a parametric model for the dependence of $\bar{\mu}(z)$ on ω_o for a vertical sun in a black sky. We will assume the same depth dependence for an ordinary sky. This is not precise, but the average cosine varies slowly and has a typical range of only about 10%. The model is:

ω_o= b / c, where b and c are the total scattering and attenuation coefficients, including water.

τ = cz, is the optical depth.

$\mu_\infty(\tau) = \bar{\mu}_\infty + (\bar{\mu}(0) - \bar{\mu}_\infty) \exp(-P_\tau \tau)$.

$\mu_\infty = -1.59\, \omega_o^4 + 1.71\, \omega_o^3 - 0.467\, \omega_o^2 - 0.347\, \omega_o + 1$

$\mu(0)$= cosine of refracted solar zenith angle

$P_\tau(\omega_o) = -0.166\, \omega_o^2 + 0.341\, \omega_o + 0.0305$

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Berwald et. al. Limnology and Oceanography 1995, vol. 40, no8, pp. 1347-1357 .